Eur. Phys. J. D **11**, 465–472 (2000)

THE EUROPEAN PHYSICAL JOURNAL D

EDP Sciences

Dependence on crystal parameters of the correlation time between signal and idler beams in parametric down conversion calculated in the Wigner representation

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Received 24 January 2000 and Received in final form 24 March 2000

Abstract. The theory of parametric down conversion within the framework of the Wigner representation has been treated recently in a series of papers using the standard model Hamiltonian. Here we take a more fundamental point of view studying the mechanism, inside the crystal, for the production of the signal and idler beams. We begin from the evolution equations for the quantum field operators, pass to the Wigner function and solve the resulting (Maxwell) equations with the use of the Green's function method. We derive the time dependence of the coincidence detection probability as a function of the parameters of the nonlinear crystal (in particular the length) the radius of the pumping beam, and the bandwidth of the filters in front of the detectors.

PACS. 42.50.Ar Photon statistics and coherence theory – 03.65.Sq Semiclassical theories and applications – 42.50.Lc Quantum fluctuations, quantum noise, and quantum jumps

1 Introduction

Parametric down conversion of light (PDC from now on) is the process which occurs when a monochromatic laser at frequency ω_0, converts into pairs of highly correlated photons at frequencies ω_s, ω_i (called signal and idler respectively), where $\omega_\mathrm{s}+\omega_\mathrm{i}=\omega_0$ [1,2]. The state of the radiated field corresponding to quantum PDC has no classical counterpart. Not only is there no solution of the classical Maxwell equations in which two beams appear fulfilling the frequency matching-condition, but also there is a very short correlation time between the conjugate beams [3,4] and a high visibility of interference patterns in joint detection experiments [5]. These properties have been used in order to test Bell's inequalities [6,7] and to show other nonclassical aspects of the down converted light [8,9]. In particular the short correlation time is taken to demonstrate the corpuscular nature of light.

In a series of articles [10–13] we have analyzed, using the Wigner representation, most of the experiments performed in the last two decades using correlated light beams produced by PDC. We have shown that, for practical calculations, the Wigner representation is as efficient as the standard Hilbert space formalism of quantum optics. Obviously, the predictions agree in the two formalisms, both being equivalent forms of quantum optics. In addition to the calculational efficiency, the Wigner function provides a picture of the experiments which is intuitively appealing because it is close to the pictures of classical optics. In fact, in the Wigner representation PDC looks similar to the classical phenomenon and the results of the experiments may appear less "mind boggling" [14].

In our previous articles we started from a model Hamiltonian, which is standard in the study of PDC [15]. It contains a nonlinear term of the form

$$\hat{H}_\mathrm{int}=\sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\hat{a}_0\hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{k}'}+\mathrm{h.c.}, \tag{1}$$

where $\hat{a}_0$ is the annihilation operator of photons from the pumping laser and $\hat{a}^\dagger_{\mathbf{k}}$, $\hat{a}^\dagger_{\mathbf{k}'}$ are creation operators of (entangled) photons. The function $f(\mathbf{k},\mathbf{k}')$ is taken as given without any attempt at deriving it. In the present article we take a more fundamental point of view and shall obtain this function from the properties of the crystal (nonlinear polarizabibity and size, in particular). In this sense our article will be analogous to articles written in the sixties [16] which were the starting point of the theory of PDC, and from which the model Hamiltonian (1) originated. The difference between our approach and those studies is that we use systematically the Wigner representation. As an application of our approach we will calculate the cross-correlation of photon counts in the two beams as a function of parameters of the nonlinear crystal and the pumping laser.

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2 The Wigner representation of PDC

To start with, let us write the quantized Maxwell equations for a material medium in which there is no free charge or current volume density:

$$\nabla \cdot \hat{\mathbf{D}} = 0, \quad \nabla \times \hat{\mathbf{E}} = -\frac{\partial \hat{\mathbf{B}}}{\partial t},$$
$$\nabla \cdot \hat{\mathbf{B}} = 0, \quad \nabla \times \hat{\mathbf{H}} = \frac{\partial \hat{\mathbf{D}}}{\partial t} \cdot \quad (2)$$

$\hat{\mathbf{E}}$, $\hat{\mathbf{D}}$, $\hat{\mathbf{H}}$ and $\hat{\mathbf{B}}$ are respectively the operators corresponding to the electric field, the electric displacement, the magnetic field and the magnetic flux density. We shall suppose a nonmagnetic medium, so that $\hat{\mathbf{B}} = \mu_0 \hat{\mathbf{H}}$, where μ_0 is the magnetic permeability of free space. On the other hand, we shall expand the polarization induced in the medium in a power series in the electric field, retaining terms to second order. Hence, the relation between $\hat{\mathbf{E}}$ and $\hat{\mathbf{D}}$ is given by

$$\hat{\mathbf{D}} = \epsilon_0 \hat{\mathbf{E}} + \hat{\mathbf{P}},$$

where

$$\hat{\mathbf{P}} = \sum_{i=1}^{3} \mathbf{u}_i \left(\epsilon_0 \sum_{j=1}^{3} \int_{-\infty}^{t} \chi_{ij}(t-t') \hat{E}_j(t') \mathrm{d}t' + 2 \sum_{j=1}^{3} \sum_{k=1}^{3} d_{ijk} \hat{E}_j \hat{E}_k \right)$$
$$= \epsilon_0 \int_{-\infty}^{t} \overline{\chi}(t-t') \hat{\mathbf{E}}(t') \mathrm{d}t' + 2 \hat{\mathbf{E}} \overline{d} \hat{\mathbf{E}}. \quad (3)$$

ϵ_0 is the permittivity of free space, and $\overline{\chi}$ and $\overline{d}$ are electric susceptibility tensors of the medium. In our study it is essential to take into account the change of refraction index with frequency and therefore we shall include retardation in the linear susceptibility. However, for the nonlinear term we shall use the (standard) approximation of neglecting retardation.

By taking into account (2) and (3) we obtain the following equation for the electric field operator in the Heisenberg picture:

$$\nabla^2 \hat{\mathbf{E}} - \frac{1}{c^2} \frac{\partial^2 \hat{\mathbf{E}}}{\partial t^2} - \frac{\partial^2}{\partial t^2} \left[\int_{-\infty}^{t} \overline{\chi}(t-t') \hat{\mathbf{E}}(t') \mathrm{d}t' \right] = 2\mu_0 \frac{\partial^2 (\hat{\mathbf{E}} \overline{d} \hat{\mathbf{E}})}{\partial t^2}, \quad (4)$$

$c = 1/\sqrt{\epsilon_0 \mu_0}$ being the speed of light in free space.

Of course anisotropy is necessary in order to account for all the properties of the PDC light, so $\overline{\chi}$ and $\overline{d}$ should be treated as tensors. However, in order to focus on the most important features of the process we shall simplify by considering an isotropic medium, thereby reducing the two tensor polarizabilities to scalars, so that the electric field always maintains the same direction. In this scalar model, which is frequently used in classical optics, equation (4) can be written in the following simplified way:

$$\nabla^2 \hat{E} - \frac{1}{c^2} \frac{\partial^2 \hat{E}}{\partial t^2} - \frac{\partial^2}{\partial t^2} \left[\int_{-\infty}^{t} \chi(t-t') \hat{E}(t') \mathrm{d}t' \right] = \beta \frac{\partial^2 \hat{E}^2}{\partial t^2}, \quad (5)$$

where we have labelled $\beta \equiv 2\mu_0 d$.

Equation (5) is an inhomogeneous wave equation in which the source of radiation is a quadratic function of $\hat{E}$. In order to solve it we shall consider an adiabatic switch on of the interaction, *i.e.* we shall consider the following equation

$$\nabla^2 \hat{E} - \frac{1}{c^2} \frac{\partial^2 \hat{E}}{\partial t^2} - \frac{\partial^2}{\partial t^2} \left[\int_{-\infty}^{t} \chi(t-t') \hat{E}(t') \mathrm{d}t' \right] = \lambda(t) \beta \frac{\partial^2 \hat{E}^2}{\partial t^2}, \quad (6)$$

$\lambda(t)$ being a slowly varying function of time, so that $\lambda(t) = 0$ at $t \to -\infty$, and $\lambda(t) = 1$ at $t \geq 0$. We shall take the state of the radiation at $t \to -\infty$, which is fixed in the Heisenberg representation, to be the corresponding to a laser, *i.e.* a coherent state $|\phi\rangle$, fulfilling:

$$\hat{E}^{(+)} |\phi\rangle = E_{\mathrm{laser}}^{(+)} |\phi\rangle. \quad (7)$$

$\hat{E}^{(+)}$ is the part of the electric field operator that only contains destruction operators:

$$\hat{E}^{(+)} = \mathrm{i} \sum_{\mathbf{k}} \left(\frac{\hbar \omega_{\mathbf{k}}}{L_0^3} \right)^{\frac{1}{2}} \hat{a}_{\mathbf{k}} \mathrm{e}^{-\mathrm{i}\omega_{\mathbf{k}} t + \mathrm{i}\mathbf{k}\cdot\mathbf{r}}, \quad (8)$$

L_0^3 being the normalization volume, and $\omega_{\mathbf{k}} = |\mathbf{k}|c$.

It is possible to use the vacuum field as the initial state if we perform the following change of variables:

$$\hat{E}^{(+)} = \hat{E}'^{(+)} + E_{\mathrm{laser}}^{(+)}, \quad (9)$$

By substituting (9) into equation (5), we have

$$\nabla^2 (\hat{E}' + E_{\mathrm{laser}}) - \frac{1}{c^2} \frac{\partial^2 (\hat{E}' + E_{\mathrm{laser}})}{\partial t^2} - \frac{\partial^2}{\partial t^2} \left[\int_{-\infty}^{t} \chi(t-t') [\hat{E}' + E_{\mathrm{laser}}](t') \mathrm{d}t' \right] = \beta \frac{\partial^2 (\hat{E}' + E_{\mathrm{laser}})^2}{\partial t^2} \cdot \quad (10)$$

Now, if we consider the situation at $t \to -\infty$ in which $\lambda = 0$, the state must fulfill the condition $\hat{E}^{(+)}|\phi\rangle = E_{\mathrm{laser}}^{(+)}|\phi\rangle$, *i.e.* $\hat{E}'^{(+)}|\phi\rangle = 0$. Hence we have shown that working with $\hat{E}(\mathbf{r}, t)$ and a coherent state as the initial state is completely equivalent to working with $\hat{E}'(\mathbf{r}, t)$ and the vacuum state.

From now on we shall write $E_{\rm laser}$ as a quasimonochromatic beam of frequency ω_0, wave vector $\mathbf{k}_0$, and radius R:

$$E_{\rm laser} = E^{(+)}_{\rm laser} + E^{(-)}_{\rm laser},$$

where

$$E^{(+)}_{\rm laser}(\mathbf{r},t) = V_0\,{\rm e}^{-\frac{x^2+y^2}{2R^2}}{\rm e}^{-{\rm i}\omega_0 t+{\rm i}\mathbf{k}_0\cdot\mathbf{r}},\quad \mathbf{k}_0 = \frac{\omega_0 n}{c}\mathbf{u}_z, \quad (11)$$

and we are taking a coordinate system $OXYZ$, O being the center of the crystal and $\mathbf{u}_z$ a unit vector in the direction of the pumping. Strictly speaking (11) is a solution of the homogeneous wave equation only in the case $R \to \infty$ [1], but we follow the usual approximation of equation (11) in order to account for the lateral size of the beam.

Let us restrict our attention to equation (10). If there were no laser beam, that is if we made $E_{\rm laser} = 0$ in equation (10), then this equation would represent the evolution of the vacuum due to the presence of the crystal, and would give rise to just a modified vacuum. If we take into account that the laser is very intense it seems reasonable to discard the term $\beta\partial^2\hat{E}'^2/\partial t^2$ from (10) because its contribution to the radiated field will be very small compared with the others. Indeed an approximation equivalent to this is standard in all treatments of PDC. Hence equation (10) reduces to the following equation, which is linear in the field operators:

$$\nabla^2(\hat{E}' + E_{\rm laser}) - \frac{1}{c^2}\frac{\partial^2(\hat{E}' + E_{\rm laser})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t}\chi(t-t')[\hat{E}' + E_{\rm laser}](t'){\rm d}t'\right] = \beta\frac{\partial^2(2E_{\rm laser}\hat{E}' + E^2_{\rm laser})}{\partial t^2}\cdot \quad (12)$$

Let us now pass to the Wigner representation. As is well-known, the evolution equations of the Wigner field amplitudes are the same as the Heisenberg equations of motion of the quantum field amplitudes, whenever these are linear. Then, in order to go to the Wigner representation we simply remove the hats in equation (12) (we shall remove also the prime in order to simplify the notation), so that

$$\nabla^2(E + E_{\rm laser}) - \frac{1}{c^2}\frac{\partial^2(E + E_{\rm laser})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t}\chi(t-t')[E + E_{\rm laser}](t'){\rm d}t'\right] = \beta\frac{\partial^2(2E_{\rm laser}E + E^2_{\rm laser})}{\partial t^2}\cdot \quad (13)$$

Because we are in the Heisenberg picture the state does not change with time and we shall use the Wigner function corresponding to the initial state, that is the Wigner function of the vacuum state

$$W_{\rm vacuum}(\{\alpha_{\mathbf{k}}\},\{\alpha^*_{\mathbf{k}}\}) = \prod_{\mathbf{k}}\frac{2}{\pi}{\rm e}^{-2\alpha_{\mathbf{k}}\alpha^*_{\mathbf{k}}}, \quad (14)$$

$\alpha_{\mathbf{k}}$ being the complex amplitude corresponding to the mode $\mathbf{k}$ of the zero-point radiation

$$E_0 = E^{(+)}_0 + E^{(-)}_0,$$

where

$$E^{(+)}_0 = {\rm i}\sum_{\mathbf{k}}\left(\frac{\hbar\omega_{\mathbf{k}}}{L_0^3}\right)^{\frac{1}{2}}\alpha_{\mathbf{k}}{\rm e}^{-{\rm i}\omega_{\mathbf{k}}t+{\rm i}\mathbf{k}\cdot\mathbf{r}};\quad E^{(-)}_0 = (E^{(+)}_0)^*. \quad (15)$$

$E^{(+)}_0$ is the positive frequency part of the vacuum field. On the other hand, from (14) it follows trivially that

$$\langle\alpha_{\mathbf{k}}\rangle = 0;\quad \langle\alpha_{\mathbf{k}}\alpha_{\mathbf{k}'}\rangle = 0;\quad \langle\alpha_{\mathbf{k}}\alpha^*_{\mathbf{k}'}\rangle = \frac{1}{2}\delta_{\mathbf{k},\mathbf{k}'}. \quad (16)$$

3 First-order perturbation theory for the calculation of the down-converted field

In order to obtain the electric field radiated by the crystal we shall use a perturbative approximation to second order in β. This is due to the fact that, as we shall see, the detection probabilities in the experiments are second-order in the coupling constant. Let us express E in the following way:

$$E(\mathbf{r},t) = E_0(\mathbf{r},t) + \beta E_1(\mathbf{r},t) + \beta^2 E_2(\mathbf{r},t). \quad (17)$$

By substituting in (13) we obtain the following system of coupled equations:

$$\nabla^2(E_0 + E_{\rm laser}) - \frac{1}{c^2}\frac{\partial^2(E_0 + E_{\rm laser})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t}\chi(t-t')[E_0 + E_{\rm laser}](t'){\rm d}t'\right] = 0, \quad (18)$$

$$\nabla^2 E_1 - \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t}\chi(t-t')E_1(t'){\rm d}t'\right] = \frac{\partial^2(2E_0E_{\rm laser} + E^2_{\rm laser})}{\partial t^2}, \quad (19)$$

$$\nabla^2 E_2 - \frac{1}{c^2}\frac{\partial^2 E_2}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t}\chi(t-t')E_2(t'){\rm d}t'\right] = 2\frac{\partial^2(E_1E_{\rm laser})}{\partial t^2}\cdot \quad (20)$$

To zeroth order in β, E_0 and $E_{\rm laser}$ are given by equations (11, 15). On the other hand, as we shall deal in this work with joint detection probabilities, in which second order terms in the field are not necessary [11], we shall calculate the field only to first order.

$$E_1^{\mathrm{PDC}(+)}(\mathbf{r},t) = \frac{-\mathrm{i}LR^2V_0}{r}\sum_{\mathbf{k}}\left(\frac{\hbar\omega_{\mathbf{k}}}{L_0^3}\right)^{\frac{1}{2}}(\omega_0-\omega_{\mathbf{k}})^2\alpha_{\mathbf{k}}^*\mathrm{e}^{-\mathrm{i}(\omega_0-\omega_{\mathbf{k}})(t-\frac{r}{c_{0k}})}$$
$$\times\exp\left\{-R^2[(\omega_0-\omega_{\mathbf{k}})\frac{x}{rc_{0k}}+k_x]^2-R^2[(\omega_0-\omega_{\mathbf{k}})\frac{y}{rc_{0k}}+k_y]^2\right\}\mathrm{sinc}\left\{\frac{L}{2}[(\omega_0-\omega_{\mathbf{k}})\frac{z}{rc_{0k}}+k_z-k_0]\right\}+\mathrm{c.c.};$$
$$c_{0k}=\frac{c}{n_{0k}}, \quad (26)$$

The radiation source of E_1 contains oscillatory terms at frequencies $\omega_0-\omega_{\mathbf{k}}$ and $\omega_0+\omega_{\mathbf{k}}$, coming from E_0E_{laser}, which give rise to parametric down conversion and parametric up conversion. It also contains frequencies 0 and $2\omega_0$, coming from E_{laser}^2, which give rise to second-harmonic generation and rectification of the input field. Since we are studying parametric down conversion, we shall take only the part of (19) corresponding to this radiation. We have:

$$\nabla^2E_1^{\mathrm{PDC}}-\frac{1}{c^2}\frac{\partial^2E_1^{\mathrm{PDC}}}{\partial t^2}-\frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t}\chi(t-t')E_1^{\mathrm{PDC}}(t')\mathrm{d}t'\right]=\sum_{\mathbf{k}}S_{\mathbf{k}}(\mathbf{r},t), \quad (21)$$

where we have defined (see Eq. (11))

$$S_{\mathbf{k}}(\mathbf{r},t)=-2\mathrm{i}V_0\,\mathrm{e}^{-\frac{x^2+y^2}{2R^2}}\left(\frac{\hbar\omega_{\mathbf{k}}}{L_0^3}\right)^{\frac{1}{2}}(\omega_0-\omega_{\mathbf{k}})^2\times\alpha_{\mathbf{k}}\mathrm{e}^{\mathrm{i}(\omega_0-\omega_{\mathbf{k}})t-\mathrm{i}(\mathbf{k}_0-\mathbf{k})\cdot\mathbf{r}}+\mathrm{c.c.} \quad (22)$$

Now, using the well-known retarded solution of the inhomogeneous wave equation, we shall obtain the radiated down converted field to first order in the following way:

$$E_1^{\mathrm{PDC}}(\mathbf{r},t)=-\frac{1}{4\pi}\sum_{\mathbf{k}}\int_V\mathrm{d}^3\mathbf{r}'\,\frac{S_{\mathbf{k}}(\mathbf{r}',t-\frac{n_{0k}|\mathbf{r}-\mathbf{r}'|}{c})}{|\mathbf{r}-\mathbf{r}'|}, \quad (23)$$

where the integration is carried over the volume of the crystal, and we have put $n_{0k}\equiv n[\omega_0-\omega_{\mathbf{k}}]$, so that the components of the radiated field will travel with different velocities inside the crystal. Here we have made the customary assumption of considering the crystal embedded in a linear medium with the same dispersion [15]. By substituting (22) into equation (23) we have:

$$E_1^{\mathrm{PDC}}(\mathbf{r},t)=\frac{\mathrm{i}V_0}{2\pi}\sum_{\mathbf{k}}\left(\frac{\hbar\omega_{\mathbf{k}}}{L_0^3}\right)^{\frac{1}{2}}\times(\omega_0-\omega_{\mathbf{k}})^2\alpha_{\mathbf{k}}\int_{-\infty}^{+\infty}\mathrm{d}x'\int_{-\infty}^{+\infty}\mathrm{d}y'\int_{-\frac{L}{2}}^{\frac{L}{2}}\mathrm{d}z'$$
$$\times\frac{\mathrm{e}^{-\frac{x'^2+y'^2}{2R^2}}\mathrm{e}^{\mathrm{i}(\omega_0-\omega_{\mathbf{k}})(t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}n_{0k})}\mathrm{e}^{-\mathrm{i}(\mathbf{k}_0-\mathbf{k})\cdot\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|}+\mathrm{c.c.}, \quad (24)$$

L being the length of the crystal. Now, supposing the observation point $\mathbf{r}$ sufficiently far from the crystal, so that we may make the following far field approximations

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|}\approx\frac{1}{r};\quad |\mathbf{r}-\mathbf{r}'|\approx r(1-\frac{\mathbf{r}\cdot\mathbf{r}'}{r^2}), \quad (25)$$

we finally obtain, after some easy algebra:

$$E_1^{\mathrm{PDC}}(\mathbf{r},t)={E_1}^{\mathrm{PDC}(+)}(\mathbf{r},t)+{E_1}^{\mathrm{PDC}(-)}(\mathbf{r},t),$$

where

see equation (26) above

is the positive frequency part of the first-order down converted field.

4 The correlation time between conjugate beams

This section is devoted to the calculation of the correlation time between conjugate beams (signal and idler photons) produced in parametric down conversion. We shall analyze the dependence of this quantity with the length of the crystal, the radius of the pumping laser, and the bandwidth of the filters. For this purpose we shall consider the simplest joint detection experiment by considering two detectors placed at positions $\mathbf{r}$ and $\mathbf{r}'$ outside the crystal. The positive frequency part of the electric field at a point $(\mathbf{r},t)$ is

$$E^{(+)}(\mathbf{r},t)=E_0^{(+)}(\mathbf{r},t)+\beta E_1^{(+)}(\mathbf{r},t)+\beta^2E_2^{(+)}(\mathbf{r},t), \quad (27)$$

where $E_0^{(+)}$ and $E_1^{(+)}$ are given by (15) and (26) respectively (for simplicity we shall write $E^{(+)}$ for $E_1^{\mathrm{PDC}(+)}$).

Let us briefly present the most important results of the theory of detection in the Wigner representation (for more details see Refs. [11,13]). First, the single detection probability at the position $\mathbf{r}$ and time t is proportional to the quantity

$$P_1(\mathbf{r},t)\propto\langle I(\mathbf{r},t)-I_0(\mathbf{r})\rangle, \quad (28)$$

$I=|E^{(+)}|^2$ being the total intensity of light and $I_0=\langle|E_0^{(+)}|^2\rangle$ the average intensity of the zero-point field. I and I_0 are not well defined if we do not specify which is the relevant frequency range involved in the sum over $\mathbf{k}$ (that range is essentially defined by the detectors). However,

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{LR^2V_0\hbar}{2r'L_0^3}\sum_{\mathbf{k}}(\omega_0-\omega_{\mathbf{k}})^2\omega_{\mathbf{k}}\mathrm{e}^{-\mathrm{i}\omega_{\mathbf{k}}t+\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}(\omega_0-\omega_{\mathbf{k}})(t'-\frac{r'}{c_{0k}})}$$
$$\times\exp\left\{-R^2[(\omega_0-\omega_{\mathbf{k}})\frac{x'}{r'c_{0k}}+k_x]^2-R^2[(\omega_0-\omega_{\mathbf{k}})\frac{y'}{r'c_{0k}}+k_y]^2\right\}\mathrm{sinc}\left\{\frac{L}{2}[(\omega_0-\omega_{\mathbf{k}})\frac{z'}{r'c_{0k}}+k_z-k_0]\right\}, \quad (31)$$

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{LR^2V_0\hbar}{2r(2\pi)^3}\int\mathrm{d}k_x\int\mathrm{d}k_y\int\mathrm{d}k_z(\omega_0-\omega_{\mathbf{k}})^2\omega_{\mathbf{k}}\mathrm{e}^{-\mathrm{i}\omega_{\mathbf{k}}t}\mathrm{e}^{ir(k_x\sin\theta+k_z\cos\theta)}$$
$$\times\,\mathrm{e}^{-\mathrm{i}(\omega_0-\omega_{\mathbf{k}})(t'-\frac{r}{c_{0k}})}\exp\left\{-R^2[(\omega_0-\omega_{\mathbf{k}})\frac{-\sin\theta}{c_{0k}}+k_x]^2-R^2k_y^2\right\}\mathrm{sinc}\left\{\frac{L}{2}[(\omega_0-\omega_{\mathbf{k}})\frac{\cos\theta}{c_{0k}}+k_z-k_0]\right\};$$
$$\frac{\omega_0}{2}-\frac{\Delta}{2}<\omega_{\mathbf{k}}<\frac{\omega_0}{2}+\frac{\Delta}{2}, \quad (33)$$

$\langle I-I_0\rangle$ is well defined because for all modes which do not take part in the detection that difference is zero.

On the other hand, in PDC experiments the joint detection probability at $(\mathbf{r},t)$ and $(\mathbf{r}',t')$ is given by

$$P_{12}(\mathbf{r},t;\mathbf{r}',t')\propto\langle\{I(\mathbf{r},t)-I_0(\mathbf{r})\}\{I(\mathbf{r}',t')-I_0(\mathbf{r}')\}\rangle. \quad (29)$$

By taking into account that the Wigner fields are Gaussian processes, and neglecting fourth-order terms in β, it can be proved that

$$P_{12}(\mathbf{r},t;\mathbf{r}',t')\propto|\langle E^{(+)}(\mathbf{r},t)E^{(+)}(\mathbf{r}',t')\rangle|^2$$
$$=\beta^2|\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle + \langle E_0^{(+)}(\mathbf{r}',t')E_1^{(+)}(\mathbf{r},t)\rangle|^2. \quad (30)$$

By taking into account equations (15, 16, 26), we have

see equation (31) above

where $\mathbf{r}'=x'\mathbf{u}_x+y'\mathbf{u}_y+z'\mathbf{u}_z$, $\mathbf{r}=x\mathbf{u}_x+y\mathbf{u}_y+z\mathbf{u}_z$, and we have labelled $r'\equiv|\mathbf{r}'|$. The second term of (30) is obtained by making the exchange $\mathbf{r}\leftrightarrow\mathbf{r}'$, and $t\leftrightarrow t'$.

Let us now consider that the detectors are placed in the directions of two conjugate beams of frequencies $\omega_0/2$ (this is the symmetrical case, usually called degenerate case), and at the same distance r from the center of the crystal. For simplicity we shall take $\mathbf{r}$ and $\mathbf{r}'$ in the plane OXZ, *i.e.* $y=y'=0$, so that

$$\mathbf{r}=r(\sin\theta\mathbf{u}_x+\cos\theta\mathbf{u}_z);$$
$$\mathbf{r}'=r(-\sin\theta\mathbf{u}_x+\cos\theta\mathbf{u}_z). \quad (32)$$

θ can be easily calculated by using the frequency-matching conditions. Its value is $\cos\theta=c_{\omega_0/2}/c_{\omega_0}$. On substituting (32) into equation (31), and replacing the sum by an integral, *i.e.* making $\sum_{\mathbf{k}}/L_0^3\rightarrow\int\mathrm{d}^3k/(2\pi)^3$, we have:

see equation (33) above

where $\omega_{\mathbf{k}}=c_{\mathbf{k}}\sqrt{k_x^2+k_y^2+k_z^2}$ and $(\omega_0/2-\Delta/2,\omega_0/2+\Delta/2)$ is the bandwidth allowed by a filter that we assume in front of the detector.

Equation (33) is the starting point for the study of the dependence of the correlation time (τ) with the different experimental parameters. The calculation of the integrals is involved, but it may be simplified if we take into account that the integrand contains the product of two highly peaked functions, namely a Gaussian and a sinc. If we assume that one of the peaks is much narrower than the other, the former may be approximated by a Dirac's delta and the calculation is easy. Consequently we shall study the following cases:

1. first, we shall consider $R\rightarrow\infty$ and $\Delta\rightarrow\infty$ in order to obtain τ as a function of L;
2. secondly we take a long crystal where we make $L\rightarrow\infty$, $\Delta\rightarrow\infty$, and we study the dependence of τ with R;
3. finally, within the long crystal approximation ($L\rightarrow\infty$), we shall analyze the dependence of τ on R and the bandwidth (Δ) of the filters placed in front of the detectors.

4.1 The short crystal case (R → ∞)

In this case, making use of the relations

$$\lim_{R\rightarrow\infty}R\,\mathrm{e}^{-R^2k_y^2}=\sqrt{\pi}\delta(k_y),$$

$$\lim_{R\rightarrow\infty}R\,\mathrm{e}^{-R^2[(\omega_0-\omega_{\mathbf{k}})\frac{-\sin\theta}{c_{0k}}+k_x]^2}=\sqrt{\pi}\delta\left[(\omega_0-\omega_{\mathbf{k}})\frac{-\sin\theta}{c_{0k}}+k_x\right],$$

integrating over k_y, and changing to polar coordinates (k_x, $k_z\rightarrow\omega_{\mathbf{k}}$, ψ),

$$k_x=\frac{\omega_{\mathbf{k}}}{c_{\mathbf{k}}}\sin\psi;\quad k_z=\frac{\omega_{\mathbf{k}}}{c_{\mathbf{k}}}\cos\psi,$$

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{LV_0\hbar}{4r(2\pi)^2}\int_0^{\omega_0}\mathrm{d}\omega_{\mathbf{k}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\mathrm{d}\psi\,\frac{\omega_{\mathbf{k}}^2}{c_{\mathbf{k}}^2}(\omega_0-\omega_{\mathbf{k}})^2\mathrm{e}^{-\mathrm{i}\omega_{\mathbf{k}}t}$$
$$\times\exp\left\{\mathrm{i}r\frac{\omega_{\mathbf{k}}}{c_{\mathbf{k}}}\cos(\psi-\theta)-\mathrm{i}(\omega_0-\omega_{\mathbf{k}})(t'-\frac{r}{c_{0k}})\right\}\delta\left[(\omega_0-\omega_{\mathbf{k}})\frac{-\sin\theta}{c_{0k}}+\frac{\omega_{\mathbf{k}}}{c_{\mathbf{k}}}\sin\psi\right]$$
$$\times\,\mathrm{sinc}\left\{\frac{L}{2}\left[(\omega_0-\omega_{\mathbf{k}})\frac{\cos\theta}{c_{0k}}+\frac{\omega_{\mathbf{k}}}{c_{\mathbf{k}}}\cos\psi-\frac{\omega_0}{c_{\omega_0}}\right]\right\}\cdot \quad (34)$$

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{LV_0\hbar\omega_0^4}{4r(2\pi)^2c\sin\theta}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\mathrm{d}\psi\,\frac{\frac{\sin^2\psi}{\sin^2\theta}}{(1+\frac{\sin\psi}{\sin\theta})^4}\frac{1}{|1+\frac{\sin\psi}{\sin\theta}|}$$
$$\times\exp\left\{\mathrm{i}\frac{\omega_0}{1+\frac{\sin\psi}{\sin\theta}}\left[t'-t+\frac{r}{c}[-1+\cos(\psi-\theta)]\right]\right\}\mathrm{e}^{-\mathrm{i}\omega_0(t'-\frac{r}{c})}\mathrm{sinc}\left\{\frac{L\omega_0\sin\theta}{2c}\frac{\cos\psi-\cos\theta}{\sin\psi+\sin\theta}\right\};\quad \psi\neq-\theta. \quad (36)$$

---

we have

see equation (34) above.

In order to make the integration in $\omega_{\mathbf{k}}$ we shall use the relation

$$\int\mathrm{d}x\delta[f(x)]g(x)=\sum_i\frac{g(x_i)}{|f'(x_i)|};\quad f(x_i)=0, \quad (35)$$

and we shall make the approximation of neglecting the variation of the velocity of light with frequency in a neighbourhood of $\omega_0/2$, *i.e.* in the rest of the paper only two velocities of light will be considered, namely the one corresponding to the laser c_{ω_0}, and $c\equiv c(\omega_{\mathbf{k}}\neq\omega_0)$. We have

see equation (36) above.

Now, by considering typical values of the experimental parameters ($L\approx10^{-3}$ m, $\omega_0\approx10^{15}$ rad/s, and $\theta\approx10°$), we have

$$\frac{L\omega_0\sin\theta}{2c}\approx10^3.$$

Hence, the sinc function in (36) is different from zero only when the matching condition $\psi\approx\theta$ is fulfilled. Due to this fact we shall make the change of variables $\psi=\theta+\epsilon$, we shall express to first order of ϵ the arguments of the exponentials and the sinc function and to zeroth order elsewhere, and the range of integration will be extended between $-\infty$ and $+\infty$. We have:

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{LV_0\hbar\omega_0^4}{128r(2\pi)^2c\sin\theta}\mathrm{e}^{-\mathrm{i}\frac{\omega_0}{2}(t+t'-\frac{2r}{c})}$$
$$\times\int_{-\infty}^{+\infty}\mathrm{d}\epsilon\,\mathrm{sinc}\left\{\frac{L\omega_0\sin\theta}{4c}\epsilon\right\}\mathrm{e}^{-\mathrm{i}\frac{\omega_0}{4}\epsilon\,\mathrm{cotan}\,\theta\,(t'-t)}. \quad (37)$$

The second term of (30) is obtained by making $\theta\leftrightarrow-\theta$, and $t\leftrightarrow t'$. After some easy calculations we arrive to the following expression for P_{12}:

$$P_{12}\propto\beta^2\left(\frac{LV_0\hbar\omega_0^4}{64r(2\pi)^2c\sin\theta}\right)^2$$
$$\times\left(\int_{-\infty}^{+\infty}\mathrm{d}\epsilon\,\mathrm{sinc}\left\{\frac{L\omega_0\sin\theta}{4c}\epsilon\right\}\cos\left[\frac{\omega_0}{4}\epsilon\,\mathrm{cotan}\,\theta\,(t'-t)\right]\right)^2. \quad (38)$$

Finally, making use of the relation

$$\int_0^\infty\mathrm{d}x\frac{\sin ax\cos bx}{x}$$
$$=\left\{\frac{\pi}{2}\ (a>b\geq0);\ \ \frac{\pi}{4}\ (a=b\geq0);\ \ 0\ (b>a\geq0)\right\}, \quad (39)$$

the resulting coincidence probability is

$$P_{12}(|t'-t|)=\frac{K\beta^2}{r^2}\Theta\left(|t'-t|-\frac{L\sin^2\theta}{c\cos\theta}\right), \quad (40)$$

K being a constant. Equation (40) defines a coherence time between conjugate beams which is given by

$$\tau=\frac{L\sin^2\theta}{c\cos\theta}=L\sqrt{\frac{c_{\omega_0}^2}{c^2}-1}, \quad (41)$$

where we have taken into account that $\cos\theta=c/c_{\omega_0}$. A similar result was obtained in [17] using the Hilbert-space formulation. By substituting typical values into equation (41) we obtain

$$\tau\approx0.1\,\mathrm{ps}. \quad (42)$$

4.2 The long crystal case ($\mathbf{L\to\infty}$)

Using the relation

$$\lim_{L\to\infty}L\,\mathrm{sinc}\left\{\frac{L}{2}[(\omega_0-\omega_{\mathbf{k}})\frac{\cos\theta}{c_{0k}}+k_z-k_0]\right\}$$
$$=2\pi\delta\left[(\omega_0-\omega_{\mathbf{k}})\frac{\cos\theta}{c_{0k}}+k_z-k_0\right],$$

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{R^2V_0\hbar\pi}{r(2\pi)^3c^3}\int_0^{\omega_0}\mathrm{d}\omega\int_0^{\pi}\sin\alpha\,\mathrm{d}\alpha\int_0^{2\pi}\mathrm{d}\psi\,(\omega_0-\omega)^2\omega^3$$
$$\times\,\mathrm{e}^{-\mathrm{i}\omega t}\exp\{\mathrm{i}r\frac{\omega}{c}(\sin\alpha\,\cos\psi\,\sin\theta+\cos\alpha\,\cos\theta)\}\exp\{-\mathrm{i}(\omega_0-\omega)(t'-\frac{r}{c})\}\delta[\frac{\omega}{c}(\cos\alpha-\cos\theta)]$$
$$\times\exp\left\{-\frac{R^2}{c^2}[(\omega_0-\omega)^2\sin^2\theta-2(\omega_0-\omega)\omega\sin\alpha\,\cos\psi\,\sin\theta+\omega^2\sin^2\alpha]\right\}. \quad (43)$$

$$\langle E_0^{(+)}(\mathbf{r},t)E_1^{(+)}(\mathbf{r}',t')\rangle = \frac{R^2V_0\hbar\pi\omega_0^5}{32r(2\pi)^3c^2}\mathrm{e}^{-\mathrm{i}\frac{\omega_0}{2}(t+t'-\frac{2r}{c})}\int_{-1}^{1}\mathrm{d}u\int_0^{2\pi}\mathrm{d}\psi\,(1-u^2)^2\exp\{-\mathrm{i}\frac{\omega_0}{2}(t-t')u\}$$
$$\times\exp\{-\mathrm{i}\frac{r\omega_0\sin^2\theta}{2c}(1+u)(1-\cos\psi)\}\exp\{-\frac{R^2\omega_0^2\sin^2\theta}{2c^2}[1+u^2-(1-u^2)\cos\psi]\}. \quad (44)$$

$$P_{12}\propto\beta^2\left[\frac{R^2V_0\hbar\pi\omega_0^5}{16r(2\pi)^3c^2}\right]^2\left|\int_{-\infty}^{+\infty}\mathrm{d}u\cos[\frac{\omega_0}{2}(t-t')u]\,\mathrm{e}^{-\frac{R^2\omega_0^2\sin^2\theta}{c^2}u^2}\int_{-\infty}^{+\infty}\mathrm{d}\psi\mathrm{e}^{\mathrm{i}\frac{r\omega_0}{4c}\sin^2\theta\,(1+u)\psi^2}\,\mathrm{e}^{-\frac{R^2\omega_0^2\sin^2\theta}{4c^2}(1-u^2)\psi^2}\right|^2. \quad (45)$$

---

and changing to spherical polar coordinates $(k_x,k_y,k_z)\rightarrow(\omega_{\mathbf{k}},\alpha,\psi)$:

$$k_z=\frac{\omega}{c}\cos\alpha;\quad k_x=\frac{\omega}{c}\sin\alpha\,\cos\psi;\quad k_y=\frac{\omega}{c}\sin\alpha\,\sin\psi,$$

where we have simplified the notation by putting $\omega_{\mathbf{k}}\equiv\omega$, we get from (33)

see equation (43) above.

Performing the integration over α (see (35)), and by making the change $\omega\equiv(\omega_0/2)(1+u)$ we get, after some algebra:

see equation (44) above.

By taking into account that the typical value of R is 10^{-3} m, we have

$$\frac{R^2\omega_0^2\sin^2\theta}{2c^2}\approx 10^5,$$

so that the values of u and ϕ whose contributions to (44) are relevant, are those nearly equal to zero (note that $u=0$ and $\psi=0$ are the corresponding values for the perfect matching). Hence, we shall make the approximation $(1-u^2)^2\approx 1$ in the second line of (44), we shall expand $\cos\psi$ to second order in ψ, and the range of integration will be extended between $-\infty$ and $+\infty$. By taking into account also the second term of (30) we get the following expression for P_{12}:

see equation (45) above.

The integration in ψ can be performed by using the following relations

$$\int_{-\infty}^{+\infty}\mathrm{d}x\,\mathrm{e}^{-bx^2}\cos(ax^2)=2\frac{\sqrt{\pi}}{(b^2+a^2)^{\frac{1}{4}}}\cos\left(\frac{1}{2}\arctan\frac{a}{b}\right),$$
$$\int_{-\infty}^{+\infty}\mathrm{d}x\,\mathrm{e}^{-bx^2}\sin(ax^2)=2\frac{\sqrt{\pi}}{(b^2+a^2)^{\frac{1}{4}}}\sin\left(\frac{1}{2}\arctan\frac{a}{b}\right),$$

where

$$a\equiv\frac{r\omega_0}{4c}(1+u)\sin^2\theta;\quad b\equiv\frac{R^2\omega_0^2\sin^2\theta}{4c^2}(1-u^2). \quad (46)$$

From (46) it can be seen that

$$\frac{a}{b}=\frac{r}{R}\,\frac{c}{R\omega_0}\,\frac{1}{1-u}\,. \quad (47)$$

We see that a/b is the product of two parameters: the first one is the ratio between the distance r from the crystal to the detector and the transversal size of the crystal (given by R); the other one is the ratio between the typical wavelength and R. By taking into account that the typical value of r is 1 m, we have $a/b\approx 0.1$, so that the integration in ψ can be approximated by $2\sqrt{\pi/b(u=0)}$. By taking into account this approximation in (45) we have

$$P_{12}\ =\ \frac{C\beta^2}{r^2}\left|\int_{-\infty}^{+\infty}\mathrm{d}u\,\cos[\frac{\omega_0}{2}(t-t')u]\,\mathrm{e}^{-\frac{R^2\omega_0^2\sin^2\theta}{c^2}u^2}\right|^2, \quad (48)$$

C being a constant. Now, by using the following relation

$$\int_{-\infty}^{+\infty}\mathrm{d}x\,\mathrm{e}^{-c^2x^2}\cos\mathrm{d}x=\frac{\pi}{c}\mathrm{e}^{-\frac{d^2}{4c^2}},$$

we finally get

$$P_{12}=\frac{C'}{r^2}\exp\left\{-\frac{(t-t')^2}{2[\frac{2\sqrt{2}R\sin\theta}{c}]^2}\right\}, \quad (49)$$

which gives us the following value for the coherence time:

$$\tau=\frac{2\sqrt{2}R\sin\theta}{c}\approx 1\,\mathrm{ps}. \quad (50)$$

4.3 Influence of a filter

Let us finally describe the situation in which a filter is placed in front of the detectors. For the sake of simplicity we shall suppose a Gaussian filter centered at $\omega_0/2$ with a bandwidth Δ:

$$f(\omega) = \exp\left\{-\frac{(\omega - \frac{\omega_0}{2})^2}{2\Delta^2}\right\} = \exp\left\{-\frac{\omega_0^2 u^2}{8\Delta^2}\right\}; \qquad \omega \equiv \frac{\omega_0}{2}(1+u), \quad (51)$$

and also that $L \to \infty$. These two considerations will allow us to study this case from the results obtained in the above section. By taking into account the filter in (48), we have

$$P_{12} \propto \left| \int_{-\infty}^{+\infty} \mathrm{d}u \cos\left[\frac{\omega_0}{2}(t-t')u\right] \times \exp\left\{-\frac{\omega_0^2}{4}\left[\frac{4R^2\sin^2\theta}{c^2} + \frac{1}{2\Delta^2}\right]u^2\right\}\right|^2, \quad (52)$$

and we obtain the following coherence time which is a function of R and Δ:

$$\tau = \frac{2\sqrt{2}R\sin\theta}{c}\sqrt{1+\frac{c^2}{8\Delta^2 R^2 \sin^2\theta}} \quad (53)$$

$$\longrightarrow \frac{1}{\Delta}, \quad (54)$$

the last limit corresponding to the case $2\sqrt{2}R\sin\theta \ll c\Delta^{-1}$, a result already obtained elsewhere [15].

5 Conclusions

In this article we have studied the production of parametric down-converted radiation starting from the quantized electromagnetic field and passing to the Wigner representation. This provides a connection between fundamental equations of quantum electrodynamics and the widely used model Hamiltonian. In fact our equation (26) is essentially the same which we used for our previous calculations [10–13] where we started from the model Hamiltonian (1), that is equation (4.10) of [10], equations (26, 27) of [11], and equations (17, 18) of [13]. The only difference is that here we have made the calculation using first order perturbation theory. The second order term, calculated elsewhere [18], is not included for the sake of simplicity.

The connection with the model Hamiltonian theory proves that our approach is able to interpret all experiments analyzed in our previous papers [10–13]. These include, in particular, "frustrated two-photon creation *via* interference" [19], "induced coherence without induced emission" [20], Franson's [21], Rarity-Tapster's [22], "dispersion cancellation" [23], "quantum eraser" [24], and various tests of Bell-type inequalities [7].

Apart from the calculational interest, the Wigner representation offers an intuitive picture of PDC which is complementary to that of the more common, but equivalent, Hilbert space formalism. Indeed the latter emphasizes the corpuscular aspects of light (*e.g.* photons are created at a point in the crystal and annihilated at a point of the detector). In contrast, the Wigner representation emphasizes the wave aspects because light is produced from a time varying polarization inside the crystal and propagates according to the (Maxwell) laws of classical wave optics.

We acknowledge financial support of CAICYT Project No. PB-98-0191 and OGICYT Project No. 191-97 (Spain).

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